# Background Lecture: Measurement, Significant Digits, and Graphing

Measurement is one of the most important aspects of working in a laboratory. All measurements give us three pieces of information, a value, a unit (or dimension), and precision or error. For example, if someone was to ask you how far it is to your home, you wouldn't just answer with a number, like 5. The person would respond with...5 what? Is it 5 feet, 5 kilometers, or 5 miles? For this reason, it is very important to specify units of measurement.

In science, we use the SI system of measurement. This system is universal in the scientific community. Some base units for this system are as follows:

distance = meter (m)

mass = kilogram (kg)

time = second (s)

temperature = Kelvin (K)

amount of substance = mole (mol)

These base units are reproducible and universally accepted. Often, however, these units are too large or too small for practical purposes. For example, one would not measure the distance from the earth to the sun in meters as it would be a ridiculously large number. By the same token, we would not measure the diameter of a bacterium in meters as it would be ridiculously small. For this reason, we use standard divisions of the base units. These are represented by prefixes as follows:

nano (n) = 10-9

micro (μ) = 10-6

milli (m) = 10-3

centi (c) = 10-2

deci (d) = 10-1

deca (D) = 10

kilo (k) = 103

mega (M) = 106

Notice that the conversions between these units are in factors of 10 so converting from one unit to another is very easy as we can simply move the decimal point the appropriate number of places either to the right or to the left. Often, however, we make the error of moving the decimal point in the wrong direction. This problem can be solved by using **dimensional analysis**. In this method, we include the unit along with the numbers in a calculation and treat it as such. The answer in our calculation will be correct in both the units and the value! For example, suppose we measured a distance of 124.89 meters and wanted to convert it to centimeters.

124.89 m = ? cm.

First we need a conversion from meters to centimeters. Notice, from the previous page, that 1 cm = 10-2 m, we can also express that as 100cm = 1m so we can use this relationship. We want the units to be changed, but not the value. We remember, from math, that if we multiply a value by 1, we do not change the value. We can express the above conversion factor as one of the following:

**Equation 1:** 100cm = 1 = 1m

1m 100cm

We now need to determine which factor to use. We want our answer to be expressed in cm. This means that cm has to be in the numerator of our conversion factor. Furthermore, we want m to cancel out so, in our conversion factor, it has to be in the denominator. We therefore would have to set up the problem as follows:

**Equation 2:** 124.89 ~~m~~ X 100cm = 12489cm

1~~m~~

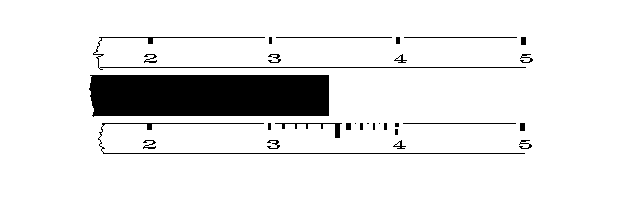
Notice how the units cancel.

This method is quite powerful and can be used for most types of calculations. It will be used throughout this course.

In making measurements it is important to record the data to the correct number of digits. In science, the measurements 5m, 5.0m and 5.00m have very different meanings. The difference lies not in their values, but in their precision. It is important to make a distinction between accuracy and precision.

Accuracy is the closeness of values to the correct value. The accuracy of a measurement describes how much error there is. Determination of error will be discussed later.

Precision, on the other hand, describes the range of error or tolerance around the measurement. All measurements have, associated with them, some degree of uncertainty. This is the tolerance associated with a measurement. To illustrate this, let us look at a measurement and the inherent error associated with it.



Look at the diagram of the rulers on this page. Notice that the top ruler is marked off every one unit. If we are to use this ruler to measure the bar next to it, we can see that it is longer than 3 units and shorter than 4 units. We can, therefore, guess that it is about 3.5 units in length. In the measurement, the 3 is certain; we know that the line is at least 3 units long, but the 5 is an estimate or guess. We, therefore, have introduced error into our measurement. It is understood that the value, 5, is really a guess that its value is between 4 and 6. In other words, it is understood that the guess is ± 1 in the guessed decimal place. Since this example has the guess in the tenths place, we have a guess of ± 0.1 units. This means that our measured value is really 3.5± 0 .1 or that the length of the bar is in the range 3.4 - 3.6 units. As long as the bar's actual length is within this range we are accurate. The precision of this measurement is ±0 .1 units.

How can we improve the precision of the measurement? Use a more precise instrument! Suppose we used the second ruler to measure the length. Notice that this ruler is marked off in increments of 0.1 units. We can now say that the bar is greater than 3.4 and less than 3.5 units long. We can estimate that it is 3.44 units long. Here, the second 4 is a guess so, since it is in the hundredth’s place, our precision is ± .01 units. therefore, our measurement is 3.44 ±.01 or 3.43 – 3.45. Notice that with both measurements we are accurate (since the correct answer lies in the range specified). However, with the new measurement, we have a narrower range. (3.43 - 3.45 v 3.4 - 3.6). The second measurement, with a precision of ± 0.01 units, is more precise.

**In general, we can estimate between any two marks on our instruments. Therefore, we record measurements to one decimal place beyond the marks on the instrument**. For example, if a ruler is marked off in centimeters, we should record to tenths of a cm. (millimeters) (.1cm). If a thermometer is marked of in 1°C increments, we would record to tenths of a degree (0.1°C) etc. Now let's go back to the original statement, the value, 5, really means the range 4 - 6, 5.0 means 4.9 - 5.1 and 5.00 means 4.99 - 5.01. This illustrates the importance of recording data correctly. It tells the reader how precisely the measurement was made. This data is summarized in the table on the next page.

|  |  |  |
| --- | --- | --- |
| Value | Precision | Range |
| 5 | ±1 | 4 - 6 |
| 5.0 | ±0.1 | 4.9 - 5.1 |
| 5.00 | ±0.01 | 4.99 - 5.01 |

In recording measurements, the number of digits in the measurement tells us the value **and** the precision of the measurement. Remember, the last value is a guess. The digits in a measurement are called **significant figures** (or significant digits). Significant figures are all of the digits in a measurement that are certain plus one that is a guess. The number of significant figures in a reported number and thus, the precision, can be determined by looking at the value. In our numbering system, all of the digits have only one meaning (their value) except zero. Zero can be either the value, 0, or a placeholder. Therefore, we must have a means to determine if a zero is significant or not. The rules for significant figures are as follows:

1. All nonzero digits are significant.

2. Zeroes between significant figures are significant.

3. Zeroes **after** a significant figure **and** after a decimal point are significant.

For example the following numbers all have three significant figures.

123, 303, 4.00, 4250, 0.00400

(Note: In the number 4250, the zero is after a significant figure but not after a decimal point. In the number 0.00400, the last three zeros are significant but not the first three by rule 3)

The precision depends on the decimal place of the last significant figure as this digit is the estimate. Unless we are otherwise told, the precision is ± 1 in the last significant decimal place. Consider the following table: (The last significant figure is in bold and underlined.)

|  |  |  |
| --- | --- | --- |
| **Number** | **#significant figures** | **precision** |
| 21**2** | 3 | ± 1 |
| 2**5**0 | 2 | ± 10 |
| 0.035**0** | 3 | ±0.0001 |

The only problem with this notation is the matter of expressing a number such as 500 but indicating 3 significant figures. The best way is with scientific notation.

500 (3 significant figures) = 5.00 X 102 error = ± 1

500 (2 significant figures) = 5.0 X 102 error = ± 10

500 (1 significant figure) = 5 x 102 error = ± 100

We now have a means of communicating error in our measurements...by using significant figures.

Since all measurements have uncertainty and we want to express all measurements and calculations to show the precision, how do we use significant figures in calculations?

**The bottom line for calculations is that we are only allowed one decimal place with uncertainty in our answer.** There are basically two types of operations, addition/subtraction, and multiplication/division. Let's explore how this works with addition/subtraction first.

Recall that when we add numbers we line up the decimal points. In this case we have to round off our answer to the highest decimal place with uncertainty. For example, if we add 36.21 and 14.8 we get the following:

**Equation 3:** 36.2**1**

+ 14.**8** (the uncertain digits are in bold)

51.**01** = 51.**0**

Notice that there is uncertainty in the hundredths place in the first value and uncertainty in the tenths place in the second value. When we add these numbers together we get uncertainty in both the tenths and the hundredths places. This violates the rule so we must round off to the tenths place as shown. Interestingly, when we add and subtract the number of significant figures can change as in the example below:

**Equation 4:** 9.**1**

+ 3.**4**

12.**5**

You can see above that we added two numbers with 2 significant figures and the result has three. However, the uncertainty is still in the tenths place. The rule is followed. Be careful!

**For multiplication and division, we simply round the answer to the number of significant figures in the number with the fewest.** For example, if we multiply 36.21 (4 significant figures) by 14.8 (3 significant figures) the answer has to be rounded to 3 significant figures as follows:

**Equation 5:** 36.2**1** X 14.**8** = 53**5**.**980** = 53**6** (uncertainty is in bold)

When a calculations involves both multiplication/division and addition/subtraction, you must keep track of the significant figures at each step and then round and the end. For example if doing a calculation that includes both subtraction and division you would keep track of the significant figures after doing the subtraction then you would do the division and round your answer to the correct number of significant figures at the end. This avoids rounding error.

**Equation 6: (**41.2**5** – 39.**4**) = 1.**85** = 0.9**343 =** 0.9**3**

1.9**8** 1.9**8**

It is important to keep these rules in mind when doing calculations as expressing a value to the correct number of significant figures is **more correct** than carrying extra digits.

It is important to know your instruments, how to read them and what their precisions are. Measurement techniques depend on the device used. The following is a summary of how various measurements are made.

**MASS**

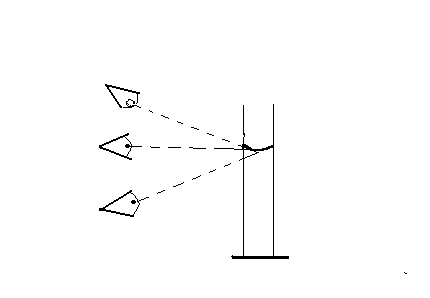
Mass is measured on a balance. The difference between a balance and a scale is that a balance always compares the mass to known masses. It is, therefore, independent of gravity. Your instructor will show you how to use the balances in your lab. In general, the number of significant figures that can be used is the number of digits in the display.

**VOLUME**

It is important to learn the uses of various containers. For example, beakers have wide mouths and spouts. They are useful for solutions that need to be stirred or will be poured into another container. Flasks have narrow openings and sloped sides. Because of this, liquids don't splash out very easily. They are excellent for swirling or adding another liquid drop wise. Graduated cylinders have precise markings so they are used for measuring liquid volumes.

Liquid volumes can be measured using a variety of containers. These types of containers vary in precision from extremely precise to rough estimates. All containers have graduated markings on the sides and the volume of the liquid is determined by reading the height of the column of liquid and comparing it to the graduations..

Since water sticks to glass, the level of the water will be a curved **meniscus**. The question that arises is, where do we read the volume? The answer is, at the **bottom** of the meniscus. It is also important to position your eye at the height of the meniscus when reading liquid volumes. This avoids what is called **parallax** error. If the eye is positioned above or below the level of the meniscus, the value read will not be the same as that read at the level of the meniscus due to the angle at which you are looking. (See the diagram below) If your eyes are below meniscus, the recorded volume will be too low. If your eyes are positioned too high, the volume will be recorded high.



**LENGTH**

Distances are measured with a ruler and, as said before, the length is recorded to one more decimal place than the ruler is marked. One other consideration is that rulers often have chewed ends. For this reason, we usually start our measurement at one mark and subtract this distance from the final value. For example, one might start at the 1.00cm mark and measure a length of 15.54cm. The actual length, therefore, is 14.54cm.

**TEMPERATURE**

Though the SI unit of temperature is the Kelvin (the absolute temperature scale), our thermometers are marked off in Celsius degrees. The conversion is straightforward as the size of a Kelvin is the same as the size of a Celsius degree. The only difference is the position of zero.

We can convert Celsius degrees to Kelvin by use of the following formula:

**Equation 7:** K = C + 273.15

(We base our rounding on the number measured on the thermometer.)

In measuring temperature, there are a few considerations. First, unlike medical thermometers, our laboratory thermometers do not need to be shaken down. To measure temperature we simply read the number off of the scale on the thermometer. It is important that we have the bulb of the thermometer completely surrounded by the item we are measuring and not in contact with anything else. For example, if we want to measure the temperature of water, we do not leave the bulb part way out of the water or touching the sides of the beaker. Also, if a temperature is changing rapidly, the alcohol in the thermometer may lag behind a bit. Therefore, one must allow time for the thermometer to equilibrate with the substance being measured before reading the temperature.

One important safety aspect is never to use a thermometer for stirring. The glass around the bulb of the thermometer is very thin and breaks easily. Stirring should be done with a glass stirring rod. Use your thermometer only to measure temperature.

**ERROR**

The final consideration we need to discuss is accuracy itself. The accuracy in a measurement is the closeness of the measured value to the correct value. The error in a measurement can be expressed as E where

**Equation 8:** E = O - A

Here O is the observed or experimental value and A is the accepted or correct value. Realize that this formula will give positive or negative errors. The sign means the direction of the error where a positive error is above the actual value and a negative error is below the actual value. Another means of expressing error is as percent error. This is expressed as below:

**Equation 9:**  % error = E X 100% = (O - A) X 100%

A A

For example suppose we experimentally determined the density of a substance to be 4.165g/mL but it actually had a density of 4.0809g/mL. The errors would be as follows:

**Example 1:**  E = O - A = 4.16**5**g/mL - 4.080**9**g/mL = 0.08**41**g/mL = 0.08**4** g/mL

% error = E X 100 % = 0.08**41**g/mL X 100 % = 2.**06082**% = 2.**1** %

A 4.080**9**g/mL

Notice that if solving for just error, then it too, is rounded to the correct number of significant figures. But when solving for percent error we want to use the rule for significant figures calculations when doing both subtraction and division. This is to keep track of significant figures and round at the end of the calculation to avoid rounding error. Also notice that the units of g/mL cancel and percent becomes the new unit.

**Drawing Graphs**

Often data is easier to understand if it is represented graphically. Graphs can be useful for seeing trends and predicting results. Most people have drawn graphs before, but not necessarily for the purpose of using the data. For this reason, in this experiment, we will review drawing graphs.

The most important thing to remember about **any** graph is that if the graph is drawn well, it needs no explanation. The graph should clearly present all of the information necessary.

Most graphs have two **axes**. For scientific purposes, the x-axis is used to represent the **independent variable.** This is the variable that is controlled by the experimenter. The y-axis is used to represent the **dependent variable**. This is the variable that depends on the independent variable. If you have trouble remembering which is which, remember, “ y depends on x” or “y is a function of x”.

In the table on the next page, there is data for the volume of a gas as a function of its Kelvin temperature. (In the temperature data, the decimal point after the number means that the zero before it is significant. All of the temperatures below have three significant figures.)

**Volume of a Gas as a function of Kelvin Temperature**

|  |  |
| --- | --- |
| **KELVIN TEMPERATURE** | **VOLUME IN LITERS** |
| 100. | 0.27 |
| 200. | 0.38 |
| 400. | 0.62 |
| 600. | 1.00 |
| 800. | 1.25 |

The way the data is presented here, the Kelvin temperature is the independent variable (x) and the volume is the dependent variable. (y). Since the graph should tell us everything, it is very important to put a descriptive title on the graph. This allows the reader to know what is actually being presented on the graph. It is also important to label the axes including the appropriate units. The blank set of axes for this data is shown on the right. Notice that each axis has both a label **and** a unit and that the title is descriptive.

All graphs should be drawn on graph paper and, as much as possible, should fill the page. For this reason, we have to **scale the axes.** This means we assign values to each line or box on the graph paper for each axis. To choose an appropriate scale, we first determine the range of values for each axis and then divide by the number of boxes. To make the graph convenient, we then round the answer  **up** to the next convenient unit. (You decide what is convenient, but make it easy for yourself. Don’t choose a weird value like 0.73 for each box!)

For example, if we had graph paper with 40 boxes and we were scaling the y-axis, we would calculate that the y-axis covers a range of

**Equation 10:** 1.25L - 0L = 1.250 liters

This gives us an assignment of

**Equation 11:** 1.25L/ 40 boxes = 0.0313 L/box

(Note that there are 3 significant figures as the number of boxes is a pure number and therefore has no uncertainty.) Since 0.0313 in not a convenient number, we might round up to 0.0500 L/box. It is important to round to a number that allows us to estimate between the values of the boxes so if we get a value that does not lie exactly on a line, we can easily estimate its location. In addition, we should round **up** to the next convenient value to keep the data on the graph. We don't want to go beyond the graph paper as that defeats the purpose of using graph paper.

Similarly, for the x-axis (if we had 40 boxes) we would get.

**Equation 12:** 800.K/ 40 boxes = 20.0 K/box (3 sf)

The graph on the next page shows a scaled set of axes for the volume/temperature data.

Notice in the axis system that the temperature has boxes every 20 K so we can go to the tens or perhaps the ones place at best. The volume axis has boxes every 0.05 L so we can probably estimate to the hundredths or perhaps thousandths place at best.

Finally, we plot the points on the graph and draw a line to best fit the points. Choose any symbol for your data points, but not one that is too small to read or one that is so large that it obliterates the points. We can then draw the best line through the data. The completed graph is shown on the next page. Notice that the line does not necessarily go through all of the points. It does, however come as close to all of the points as possible.

Now that we have a line drawn, we can make predictions about what the data would look like at specific points within or beyond our data. For example, using our graph we could get the temperature at 0.80L by drawing a line across from 0.80 L on the y-axis to the line from our data. We would then drop a vertical line down to the x-axis giving us a value of about 480K. We could use a similar process to find a volume value from a given temperature

To get information about data beyond our graph, we have to determine the equation of the line. Remember that any line can be expressed as:

**Equation 13:** y = mx + b

Where x and y are coordinates of a point on the line, m is the slope of the line and b is the y-intercept. We can calculate the slope of the line, m, by using any two points **on** **the line**. (It is important that points used are actually those on the line.. These may or may not be actual data points! In addition, try to pick points that fall on the corner of a block of the graph paper so you don't have to estimate**. Most importantly...draw the graph first!!!)**

Remember that the slope of the line is given by the following relationship:

**Equation 14:** slope = m = (y2 - y1)

(x2 - x1)

where two points on the line are (x1, y1) and (x2, y2). For example, if we use our data, we can see that two points **on the line** are: (480.K, 0.80L) and (700.K, 1.10L). (See the graph.) If we plug these into the above equation, we get a slope of:

**Equation 15:** m = (1.1**0** - 0.8**0**) L = 0.3**0** L = 0.001**364** L/K

(70**0**. - 48**0**.)K 22**0** K

= 0.001**4** L/K

(Notice that the units are included and we have expressed the slope to the correct number of significant figures.)

We can now use the slope of the line and any point on the line to calculate the intercept, b as follows:

y = mx + b

**Equation 16:** b = y - mx

If we use our slope, 0.0014L/K and a point on the line (480.K, 0.80L) we can solve for the intercept as follows:

**Equation 17:**  b = 0.8**0**L – ((0.001**4**L/K)(48**0**.K))

= 0.1**3**L

This gives us an equation of the line as follows:

**Equation 18:**  y (L) = (0.001**4** L/K) x (K) + 0.1**3**(L)

(Here we've put the units in parentheses) Or, using descriptive variables where V = volume and T = temperature.

**Equation 19:** V(L) = 0.0014(L/K)T(K) + 0.13(L)

Where V and T are the volume and temperature respectively. Notice here that we have the slope and intercept expressed to the correct number of significant figures. In addition we have units on these values. Finally, we have used descriptive variable names. (Instead of y and x, we are using V and T for volume and temperature.) All of these are good habits to create. We can now use this equation to solve for data beyond our graph.

**Example 2:**  To calculate the volume at 22**3**0K:

V(L) = 0.001**4**(L/K)T(K) + 0.1**3**(L)

here, T = 22**3**0 K so

= (0.001**4**(L/K) (22**3**0(K)) + 0.1**3**(L)

= 3.**12**L + 0.1**3**L = 3.**25**L

= 3.**3**L

(Again, notice the correct use of units and significant figures.)

**Example 3:**  To calculate the temperature at 5.16L

V(L) = 0.001**4**(L/K)T(K) + 0.1**3**(L)

here, V = 5.1**6** L so

rearranging:

(V - 0.1**3**L) = T = (5.1**6**L - 0.1**3**L) = 5.0**3**L = 3**6**00K

0.001**4**(L/K) 0.001**4**(L/K) 0.001**4** (L/K)

In this experiment, you will explore error in measurement and familiarize yourself with some of your equipment. In addition, this lab is about recording data correctly. Finally, there is some enclosed data that you are to graph. Graph paper is included. **YOU CAN NOT USE COMPUTER GENERATED GRAPHS!!!** Be sure to label all axes and put titles on your graphs. Also, be sure to do all calculations to the correct number of significant figures. Put units and descriptive variable names on your equations.